# A very short anthology of some beautiful numbers, mathematical terms & expressions

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**Armstrong Numbers**: An $n$-digit number that is the sum of the $n$th powers of its digits is called an Armstrong number. It is also sometimes known as an $n$-narcissistic number or a perfect digital invariant, or a plus perfect number. The largest Armstrong number is 115132219018763992565095597973971522401. Some examples are $153 = 1^3 + 5^3 + 3^3$, $9474 = 9^4 + 4^4 + 7^4 + 4^4$ and $54748 = 5^5 + 4^5 + 7^5 + 4^5 + 8^5$.

**Aronson's sequence**: Aronson's sequence is an integer sequence defined by the English sentence "T is the first, fourth, eleventh, sixteenth, ... letter in this sentence." Spaces and punctuation are ignored. The first few numbers in the sequence are: $1, 4, 11, 16, 24, 29, 33, 35, \ldots$.

**Astonishing numbers**: Astonishing Number is a number $N$ whose representation can be decomposed into two parts, $a$ and $b$, such that $N$ is equal to the sum of the integers from $a$ to $b$ and $a + b = N$ where $+$ denotes concatenation. For example $1 + 2 + 3 + 4 + 5 = 15$ and $2 + 3 + 4 + 5 + 6 + 7 = 27$.

For every $a = \dfrac{2.10^n - 5}{15}$ and $b = \dfrac{8.10^n - 5}{15}$, $N$ is an astonishing number.

**Autobiographical numbers**: These numbers are natural numbers with atmost 10 digits in which the first digit of the number (from left to right) tells you how many 0s there are in the number, the second digit tells you the number of 1's, the third digit tells you the number of 2's and so on.

Here is the full set of autobiographical numbers:

$$1210, 2020, 21200, 3211000, 42101000, 521001000, 6210001000.$$

**Brun's constant**: In number theory, Brun's theorem states that the sum of the reciprocals of the twin primes (pairs of prime numbers which differ by 2) converges to a finite value known as Brun's constant, usually denoted by $B_2 = 1.90216054\ldots$.

**Eddington Number**: In astrophysics, the Eddington number, $N_{Edd}$ is the number of protons in the observable universe. The term is named for British astrophysicist Arthur Eddington, who in 1938 was the first to propose a value of $N_{Edd}$ and to explain why this number might be important for physical cosmology and the foundations of physics.

**Cycling Numbers:** Take the number, 1781. Notice that $17^2 + 81^2 = 6850, 68^2 + 50^2 = 7124, 71^2+24^2 = 5617, 56^2+17^2 = 3425, 34^2+25^2 = 1781$ and we get back our original number. Such numbers are called cycling numbers. Other examples include 7141: $\{7141 \longrightarrow 6722 \longrightarrow 4973 \longrightarrow 7730 \longrightarrow 6829 \longrightarrow 5465 \longrightarrow 7141\}$.

**Durable Palindromic Primes**: If removal of the first and the last digit of a palindromic prime leaves another prime and repetitions of the digit removal operation continue to produce primes until a single digit prime is reached, the largest prime in such a sequence is said to be parent durable palindromic prime. For example, $11311.11311, 131, 3$ are primes. There are exactly ten parent durable palindromic primes.

**Exotic Numbers**: Exotic numbers are numbers that can be expressed using each of its own digits in any order only once using any mathematical symbols. For example, $715 = (7-1)!-5, 120 = ((1+2)!-0!)!$, etc. are exotic numbers.

**Filzian number**: A Filzian number is a positive integer that is equal to sum of its digits times the product of its digits. For instance, $1\times1 = 1, (1+3+5)\times1\times3\times5 = 135$ and $(1+4+4)\times1\times4\times4 = 144$. (A good exercise is to try to prove that they are the only such numbers.)

**Fortuitous numbers**: The numbers that are equal to the product of the lengths of the words in its name are called fortuitous numbers. For example, 84,672 is sounded and read "EIGHTY FOUR THOUSAND SIX HUNDRED SEVENTY TWO". If we count the letters in each of those words, and then multiply the counts, we get $6 \times 4 \times 8 \times 3 \times 7 \times 7 \times 3 = 84,672$.. Other examples are $333,396,000$, $23,337,720,000, 19,516,557,312,000, 56,458,612,224,000$, and $98,802,571,392,000$.

**Hilbert number**: The number, $2^{\sqrt{2}}$ is called Hilbert number. Gelfond-Schneider theorem states that, if $a$ and $b$ are algebraic numbers with $a \neq 0, 1$, and $b$ irrational, then any value of $a^b$ is a transcendental number. Therefore $2^{\sqrt{2}}$ is a transcendental number.

**The Hofstadter-Conway 10,000 dollar sequence**: It is defined as follows: $a(1) = a(2) = 1, a(n) = a(a(n-1)) + a(n-a(n-1)),\ n \geq 3$. The first few terms of this sequence are $1, 1, 2, 2, 3, 4, 4, 4, 5, 6, 7, 7, 8, 8, 8, 8, \ldots$.

This sequence acquired its name because John Horton Conway offered a prize of 10,000 dollars to anyone who could demonstrate a particular result about its asymptotic behaviour. The prize, since reduced to 1,000 dollars, was claimed by Collin Mallows. Douglas Hofstadter later claimed he had found the sequence and its structure some 10-15 years before Conway posed his challenge.

**Kaprekar's Constant**: 6174 is known as Kaprekar's constant after the Indian mathematician D. R. Kaprekar. This number is notable for the following rule:

- Take any four-digit number, using at least two different digits (leading zeros are allowed).
- Arrange the digits in descending and then in ascending order to get two four-digit numbers, adding leading zeros if necessary.
- Subtract the smaller number from the bigger number.
- Go back to the second step and repeat.

The above process, known as Kaprekar's routine, will always reach its fixed point, 6174, in at most 7 iterations. The only four-digit numbers for which Kaprekar's routine does not reach 6174 are repdigits such as 1111, which give the result 0000 after a single iteration. All other four-digit numbers eventually reach 6174 if leading zeros are used to keep the number of digits at 4.

**Mills' constant**: In number theory, Mills' constant is defined as the smallest positive real number $A$ such that the floor function of the double exponential function $\lfloor A^{3^n} \rfloor$ is a prime number for all natural numbers $n$. This constant is named after William H. Mills who proved in 1947 the existence of $A$ based on results of Guido Hoheisel and Albert Ingham on the prime gaps. Its value is unknown, but if the Riemann hypothesis is true, it is approximately $1.3063778838630806904686144926\ldots$. The first four Mills' primes are $2, 11, 1361$ and $2521008887$.

**Palindromic Numbers**: A palindromic number (also known as a numeral palindrome or a numeric palindrome) is a number (such as 16461) that remains the same when its digits are reversed. Interestingly, one can calculate the no of such numbers, it is $\leq 10^n = 2(10^{n/2} - 1)$ if $n$ is even and $2 \times |(10^{(n-1)/2} - 1)| + 9 \times (10^{(n-1)/2})$ if $n$ is odd.

**Ramsay Number**: The Ramsey number $R(m, n)$ gives the solution to the party problem, which asks the minimum number of guests $R(m, n)$ that must be invited so that at least $m$ will know each other or at least $n$ will not know each other. By symmetry, $R(m, n) = R(n, m)$.

**Truncatable primes**: A left-truncatable (resp. right-truncatable) prime is a prime number which, in a given base, contains no 0, and if the leading ("left") (resp. last ("right")) digit is successively removed, then all resulting numbers are prime. For example, 9137 is left-truncatable, since $9137, 137, 37$ and 7 are all prime. Another such number is 357686312646216567629137. 7393 is an example of a right-truncatable prime, since $7393, 739, 73$, and 7 are all prime. A left-and-right-truncatable prime is a prime which remains prime if the leading ("left") and last ("right") digits are simultaneously successively removed down to a one or two digit prime. 1825711 is an example of a left-and-right-truncatable prime, since $1825711, 82571, 257$ and 5 are all prime. In base 10, there are exactly 4260 left-truncatable primes, 83 right-truncatable primes, and $920, 720, 315$ left-and-right-truncatable primes.