Starting out with Olympiads

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This guide aims to get you started on your Olympiad journey. I will try to write about various resources and give my opinions on a few frequently asked questions. Of course, the answers and the resources I write about will be biased towards my own experiences. To organise the text and make it look more appealing, I will split it into answers and questions.

1. What are Math Olympiads?

Math Olympiads are competitive exams in elementary mathematics where contestants solve a small number of difficult problems in 3-6 hours, the duration can vary, but it is usually at least 4 hours. The International Mathematical Olympiad is a two-day contest in elementary mathematics where each day, the contestants have 4.5 hours to solve three problems (so, in total 9 hours to solve 6 problems!). Securing a non-zero score in such a contest is quite an accomplishment. You may familiarise yourself with the following contests.

- RMO - Regional Mathematical Olympiad
- IOQM - Indian Olympiad Qualifier in Mathematics (previously PRMO)
- INMO - Indian National Mathematical Olympiad
- IMOTC - International Mathematical Olympiad Training Camp
- TST - Team Selection Tests (these are IMO styled tests that are held to select the Indian IMO team in IMOTC)
- IMO - International Mathematical Olympiad

Visit `https://Olympiads.hbcse.tifr.res.in/` to know more and keep yourself updated regarding Indian Olympiads. Knowing how to use the Google search engine is crucial for any kind of exam preparation. The internet is going to be the handiest tool in your Olympiad journey.

2. I just got to know about Olympiads, how do I get started?

Well, first of all, welcome to the club!

- The most important thing right now, is to get an idea of what position you are at and gain some exposition. Don't directly aim for INMO, that's just stupid, you need to start from the bottom and move up the ladder. I suggest looking at a few past problems from these Olympiads. Presumably, you will see that you can solve none of them. Don't get demotivated by this. If you are an olympian, it's a part of life.
- Next thing you try to do is, complete school level math curriculum till Class 10 (Classes 11 and 12 are not needed), NCERT textbooks work; no need to do topics like coordinate geometry and trigonometry for now. If you are a beginner, it will be super helpful to have someone to address your doubts or teach topics that you find difficult. No need to do reference books like R.D. Sharma. Remember, you are trying to learn math, not prepare for your board exams.
- **Never** try to memorise theorem statements or their proofs. Quoting Evan Chen: "surface-level understanding is greatly penalised!" If you forget some fact, it's totally fine, just look it up again or rederive it yourself. You are expected to have a better understanding of your school curriculum than your friends at school, which is very much doable.
- Look at a few past problems from beginner Olympiads (like Assam Mathematics Olympiad, etc.) and their solutions. Try to properly understand them. Start a proper Olympiad book from subsection 8.1. These books will seem hard but they will get more and more doable in course of time. When you read solutions, you will ask "How was I supposed to think of that?", which is perfectly fine and understandable. With experience, you will know how to answer this (very important) question by yourself.
- **You need to spend time trying problems**. I can't stress this enough. Simply reading solutions will do nothing. You need to learn to spend time on problems but also be careful not to spend too much time; try to spend at least 20-30 minutes on each problem and don't put more than 1-1.5 hours without any progress. Experienced olympians can spend many hours or even days on a single problem. In my prime time, the average number of problems I solved per day was perhaps around 0.5-1; and no, I wasn't fooling around all day or anything, I spent the majority of my whole day on solving problems.
- With more and more experience, your understanding of basic concepts will deepen. Mathematical concepts will start making more sense than they ever made. This probably doesn't make sense now, but consider the following problem.

  Problem (USAMO 2014/3)**.** *Prove that there exists an infinite set of points*

  $$\ldots,\ P_{-3},\ P_{-2},\ P_{-1},\ P_0,\ P_1,\ P_2,\ P_3,\ \ldots$$

  in the plane with the following property: For any three distinct integers a, b, and c, points P_a, P_b, and P_c are collinear if and only if $a + b + c = 2014$.

To understand the problem, you don't need to know any math. To solve it, you only need to be familiar with polynomials (I'm sure you don't mind this problem getting spoilt because otherwise, you wouldn't be reading this article). *Technically*, anyone who knows what polynomials are should be able to solve it. But practically, you need a much deeper understanding of polynomials to solve it. This sense of deeper understanding of concepts only comes with experience and mathematical maturity.
For experts: I am aware that elliptic curves provide huge motivation towards the solution, but I can assure you that most contestants didn't know about elliptic curves, yet they still managed to solve it.

- Paraphrasing (and slightly modifying) Evan Chen's words: Math competitions are notoriously difficult. Getting even 1 problem correct in a contest like RMO/INMO is already an accomplishment. You will be competing against some students who have been seriously preparing for many years. Even then, many of these students will not do well; like in real life, hard work does not automatically guarantee success. Just like in Indian cricket, a lot of players are deserving to play international cricket, but not everybody gets to be in the playing eleven. If this sounds discouraging, it isn't meant to be. What it means is that you have to enjoy the process of learning itself, not merely a means to an end.

3. Is it too late to start in class x?

If you are in class x where $x \leq 11$, then definitely no, it's not late. You have plenty of time. Though, starting as early as possible is always better. Class 12 guys, don't get demotivated, I know a few people who started preparation for INMO in class 12 and succeeded. So, if you are in class 12, I won't tell you to not prepare, instead, I would suggest you also consider studying for ISI (Indian Statistical Institute) and CMI (Chennai Mathematical Institute) entrances, which are way easier than RMO/IOQM or INMO. Unfortunately, I have never prepared for these entrances so I won't be able to say much; but yes, look at past papers, get the book called TOMATO, research online, and figure out what you need to do.

4. How did I manage to get so much time for math contests?

The short answer is: by dropping activities that are not important to me. I had low attendance in school from childhood, and this just aggravated from the time I started studying for Olympiads. All of my past class teachers complained about my low attendance. Fortunately, I cleared INMO in class 10, so my direct selection at ISI, CMI and IIT Bombay was basically confirmed. Hence, I could afford to freely study math and not prepare for any kind of exams like JEE (or NEET) in my higher-secondary years; I wasn't interested in anything other than math anyway. And by the way, matriculation and higher school leaving exams are massively overrated; I got 95.6% and 95.4% in them, respectively, by just preparing for one month each. I understand that not everybody can afford to make the decisions I made and I am not saying you should do the same as me. Let my story

not mislead you into thinking that it is not possible to prepare for INMO together with another contest. I know many people who achieved a sub 100 rank in JEE Advanced in addition to clearing INMO. But I'd like to stress on the advice that **drop activities that are not important to you**. School curriculum is not designed for exceptional students. Not to say anything about the atrocious math curriculum in higher secondary school.

5. Why should I study for Olympiads? What are the benefits?

I don't know how to answer this question satisfactorily, perhaps because I have never asked it myself. If you like math, you shouldn't need an answer. If you are into math, I suggest that you start your Olympiad journey and see if you like it. Once you love what you do, you don't need an answer. To name some *byproducts* of succeeding in INMO: direct selection in CMI, ISI in IIT Bombay BS in Math, and a huge head start in higher mathematics (which you anyway get if you are experienced in Olympiad maths). I am not saying it is wrong to study for Olympiads with a goal in mind, but if that's your only motivation, it's an issue.

6. I can solve a few RMO problems. How do I prepare for INMO?

Congratulations! Only a few people can solve RMO problems. You always have to keep upping your level. Do past INMO question papers. Solve hard problems. Try problems from the following sources.

- Junior Balkan Mathematical Olympiad
- Balkan Mathematical Olympiad
- USAJMO
- European Girls' Mathematical Olympiad
- IMO Shortlists A1/A2, N1/N2, G1/G2/G3, C1/C2.
- Canadian Mathematical Olympiad
- All Russian Mathematical Olympiad
- Brazilian Mathematical Olympiad

7. Am I guaranteed to succeed in X contest by doing Y books and Z problems and devoting W hours daily?

No.

8. Books and resources

Most of the books can be downloaded in PDF format; you just need to do some searching ;) note that you don't have to do all the books from a particular category. Most of your time is going to be spent trying problems. The amount of time you are going to spend learning theory is negligible.

8.1. Beginner

The following books are meant to be introductions to Olympiad mathematics. I don't expect you to complete any of these books. Even doing 3-4 chapters as a beginner is impressive.

- Pranesachar and Krishnamurthy, Challenge and Thrill of Pre-College Mathematics
- Fomin, Itenberg and Genkin, Mathematical Circles
- Zeitz, The Art and Craft of Problem Solving

8.2. Intermediate

This is the learning + solving phase, hence the number of books.

- General
  - Engel, Problem Solving Strategies (except for the geometry portion)
- Algebra
  - Andreescu and Feng, 101 Problems in Algebra From the Training of the USA IMO Team
  - Wu, Functional Equations
    `https://artofproblemsolving.com/community/c6h1592427p10968935`
- Combinatorics
  - Andreescu and Feng, A Path to Combinatorics for Undergraduates
  - Soberón, Problem-Solving Methods in Combinatorics: An Approach to Olympiad Problems
- Geometry
  - Chen, Euclidean Geometry in Mathematical Olympiads
- Number Theory
  - Khurmi, Modern Olympiad Number Theory
    `https://artofproblemsolving.com/community/c6h2344755`
  - Stevens, Olympiad Number Theory Through Challenging Problems [1]
  - Andreescu, Andrica and Feng, 104 Number Theory Problems: From the Training of the USA IMO Team

[1] `http://s3.amazonaws.com/aops-cdn.artofproblemsolving.com/resources/articles/Olympiad-number-theory.pdf`

At this stage, you should have a good understanding of common methods, basic heuristics, and have developed some intuition. For example,

- Number Theory: Good understanding of modular arithmetic, Fermat's little theorem, Euler's totient function, $p-$adic valuation, Lifting the exponent, Vieta jumping, quadratic reciprocity, Pell's equation, etc.
- Geometry: Angle chasing, cyclic quadrilaterals, power of a point, homothety, bashing with complex numbers and barycentric coordinates, spiral similarity, inversion, projective geometry/homography, etc
- Algebra: Roots and coefficients, solutions of recursions, Cauchy functional equations, common functional equation "tricks" (I usually don't like associating the word "tricks" with math Olympiads, but for functional equations, it's the most appropriate), basic inequalities, etc.
- Combinatorics: Bijections, double counting, generating functions, complex number methods, graph theory, invariants, recursions, etc.

8.3. Advanced

Warning: These are well beyond the level of INMO.

At this stage, you'd most likely know what to do. Just for completeness, here's a list of things you can look up. More than doing books, you would be spending most of your time on problems now, especially IMO Shortlists.

- General
  - Dospinescu and Andreescu, Problems From the Book
- Algebra
  - Basic differential calculus, Lagrange Multipliers, calculus intuition, convex/concave functions
- Geometry
  - Zveryk, The Method of Moving Points
    `https://artofproblemsolving.com/community/c6h1884540p12835147`
  - Chroman, Goel and Mudgal, The Method of Animation
    `https://artofproblemsolving.com/community/q1h1952595p13480666`
  - You can also check out my notes on some relatively obscure but useful configurations, which may possibly be a little too elementary for you.
    `https://www.cmi.ac.in/~ayannath/oly-geo-confs.pdf`
- Combinatorics
  - Sriram, Olympiad Combinatorics

https://drive.google.com/file/d/1sQtirXxkEfWYuGSKDZ-d7VGYkR_idebY/view?usp=sharing
https://artofproblemsolving.com/community/c6h601134p12270859
  - Linear Algebra in combinatorics
  - Combinatorial Nullstellensatz
- Number Theory
  - Finite fields, algebraic numbers/integers and minimal polynomials
  - You can also check out my own handout on Analytic Number Theory which is published as a three part series in previous issues of *Ganit Bikash*.

You will have a deep understanding of universal notions like global/local, free, rigid, process, etc. Further, INMO problems should be a piece of cake.

9. Useful sites

- Art of Problem Solving (AoPS): https://aops.com/community This is very important. Make an account right now! This site is going to be your biggest friend in your journey, I highly recommend you to explore the site, especially the Contest Collections: all the past papers will be there. Don't be shy to post questions and solutions, as Cunningham's Law states "the best way to get the right answer on the internet is not to ask a question; it's to post the wrong answer".
- Evan Chen's homepage: https://web.evanchen.cc

10. Signing off

Feel free to reach me via email or AoPS. I prefer email as I may be late on replying on AoPS.

- Website: https://www.cmi.ac.in/~ayannath
- AoPS username: ayan.nmath
- Email: xy[at]cmi[dot]ac[dot]in where x is ayan and y is nath.

I welcome all kinds of queries. Good luck!